Summary of the Bayes Net Formalism

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Bayesian Networks

Two components:

1. Directed Acyclic Graph (DAG)
   - **G**: There is a node for every variable
   - **D**: Some nodes have arrows btw. them ($X \rightarrow Y$)
   - **A**: Can’t get from one node back to itself by following arrows

2. Joint Probability Distribution
   - For all $\{X, Y, \ldots, Z\}$, $P(X=x, Y=y, \ldots, Z=z)$ is defined
Example of a Bayes Net

- Directed Acyclic Graph:
  - Air Pressure
    - Barometer
    - Storm Tomorrow

- Joint Probability Distribution:
  - \( P(\text{AP}=\text{High}, \ B=\text{High}, \ ST=\text{Yes}) = 0.2 \)
  - \( P(\text{AP}=\text{High}, \ B=\text{Low}, \ ST=\text{Yes}) = 0.005 \)
  - ...
Connecting the Graph & JPD

- Markov assumption:
  “X is (probabilistically) independent of its (graphical) non-descendants conditional on its (graphical) parents.”
- “No edge ⇒ Conditional independence”
Connecting the Graph & JPD

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  “X is (probabilistically) independent of its (graphical) non-descendants conditional on its (graphical) parents.”
  “No edge $\Rightarrow$ Conditional independence”

- Example: $X \rightarrow Y \rightarrow Z$ $\Rightarrow$ $X \perp\!\!\!\!\!\!\!\!\!\!\!\perp Z \mid Y$
The Markov assumption implies a factorization of the JPD based on the graph:

\[ P(X=x, Y=y, \ldots, Z=z) = \prod P(v \mid \text{parents}(v)) \]

The Markov assumption allows us to move from graph to probability distribution.
Connecting the Graph & JPD

- Faithfulness assumption: “The (probabilistic) effects of (graphical) paths never exactly offset.”
  - “Conditional independence $\Rightarrow$ No edge”

- The Faithfulness assumption allows us to move from probability distribution to graph
Bayesian Network Example

1. Running causes you to eat more
2. Eating more causes you to gain weight
3. Running increases your metabolism
4. Increased metabolism leads to weight loss

- Note: Faithfulness rules out “Running ↑↑ Weight”
Learning Bayes Nets

● Given some data from the world, why would we want to learn a Bayes net?

1. Compact representation of the data
   ● There are fast algorithms for prediction/inference given *observations* of the environment

2. Causal knowledge
   ● There are fast algorithms for prediction/inference given *interventions* in the environment
Bayesian Updating

- Start with a probability distribution over possible graphs

$P(G)$
Bayesian Updating

- Start with a probability distribution over possible graphs, then figure out which graph is most likely given the observed data.

\[ P(G) \]
Features of Bayesian Updating

● Advantages:
  1. Output is fine-grained probabilistic information
  2. “Rational” basis for the learning algorithm
  3. Robust to data errors

● Disadvantages:
  1. Number of possible graphs is super-exponential, & likelihood functions not always solvable analytically ⇒ almost always use heuristics
  2. Cannot easily incorporate unobserved variables
Constraint-Based Learning

1. Determine the (un)conditional associations and independencies in the data
2. Determine the set of Bayes nets that could have produced that data
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X & \perp W \ (\text{all}) \\
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Features of C-B Learning

- Advantages:
  1. Feasible, asymptotically correct algorithms (though worst case requires exponential comp.)
  2. Easily incorporates unobserved variables
  3. Gives exactly the information in the data

- Disadvantages:
  1. Susceptible to mistaken independence judgments (big problem for small datasets)
  2. Cannot use fine-grained prior knowledge
Layers of “Bayes Nets”

- Start with simple association graph
Layers of “Bayes Nets”

- Then we can add a causal interpretation:
Layers of “Bayes Nets”

- Or we can provide parameter information:
Layers of “Bayes Nets”

- Or we can combine causation & parameters
Useful Websites

Tutorials:
http://www.cs.berkeley.edu/~murphyk/Bayes/bayes.html

Constraint-based learning software:
http://www.phil.cmu.edu/tetrad/index.html (free)

Bayesian learning software:
http://www.hugin.com (commercial)
http://research.microsoft.com/~dmax/winmine/tooldoc.htm (free)